Free Rising Spheres Do Not Obey Newton's Law for Free Settling

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One of the Newton's important studies was on the laws governing free settling of particles in fluid, especially in air. He measured the terminal velocity of particles with different diameters and densities dropped from the dome of St. Paul's Cathedral in London (Newton, 1760). The relationships among the particle drag, its diameter, and terminal velocity were determined. The balance of the forces applied to a settling sphere can be expressed by the following equation:

$$U_t = \sqrt{\frac{4gd(\rho_p - \rho_l)}{3\rho_l C_D}} \tag{1}$$

The drag coefficient C_D in Newton's work corresponds to a value of 0.5. The effects of diameter and density on the drag coefficient of spheres settling in Newtonian fluids of different densities and viscosities were quantified later (Allen, 1900; Lapple and Shepherd, 1940; Bailey, 1974). The well-known standard drag curve (Lapple and Shepherd, 1940; Clift et al., 1978) describes the drag coefficient as a function of the major physical parameters of spherical particles and different fluids. C_D in the Newtonian region ($10^3 < Re_t < 2 \times 10^5$) was found to be a little smaller than that obtained originally by Newton and is equal to $0.44 \pm 10\%$. Many correlations of the standard drag curve have been proposed (Clift et al., 1978; Flemmer and Banks, 1986; Turton and Levenspiel, 1986) with that of Turton and Levenspiel being perhaps the best, combining simplicity, exactness and wide-range applicability:

$$C_D = \frac{24(1 + 0.173\ Re_t^{0.657})}{Re_t} + \frac{0.413}{1 + 16{,}300\ Re_t^{-1.09}} \tag{2}$$

Unfortunately, the standard drag curve does not fit the experimental data at very high Reynolds numbers and certain density ratios, ρ_p/ρ_l (Christiansen and Barker, 1965; Clift et al., 1978). Under these conditions, the particle shows a secondary motion in nonvertical direction and rotation. As a result, a rocking and spiral trajectory is observed. The drag coefficient is larger than that predicted by the standard drag curve. This is due to the effect of the growth and shedding of turbulent eddies on the particle movement. The same effect was also observed by Newton. He payed a great deal of attention to it, but was unable to explain this effect because fluid turbulence was unknown at that time.

The free rise of spherical particles with densities smaller than that of the fluid (in this work called "light particles") is believed to obey the laws of free settling, since the same forces are applied to the particle but in the opposite directions. For example, Clift et al. (1978) in the classic monograph, *Bubbles, Drops and Particles*, assumed that the movements of free-rising and free-settling particles should be identical (except for direction, of course). This assumption, however, was not proved by experimental data.

Many authors compared the behavior of heavy settling particles with that of rising gas bubbles (Mersmann, 1978; Wallis, 1969; Perry and Chilton, 1973). Levich (1962), in his monograph, *Physicochemical Hydrodynamics*, compares the velocity of rise of air bubbles in liquids with the free-falling velocity of glass beads. However, no comparison between the behavior of gas bubbles and free rising light solid spheres has ever been made.

We have recently performed an extensive literature survey (starting from the works of Newton and finishing in 1991) on the hydrodynamics of the free rise of light solid particles. We were surprised to find almost no data reported on this subject. Allen (1900) studied rising paraffin particles in aniline at $Re_t < 30$ and found no difference in the movement of light and heavy spheres. Scoggins (1964) observed the free rise of spherical meteorological balloons. They showed a more or less spiral trajectory under very stable atmospheric conditions (Figure 1). This type of trajectory was explained by the effect of wake shedding at high Reynolds numbers. Unfortunately, the article did not contain data about the balloon's mass, and it was not possible to calculate the drag coefficient.

Recently, hydrodynamic investigations of a newly invented multiphase system, the inverse fluidized bed, were performed (Karamanev and Nikolov, 1992). The inverse fluidized bed consists of fluid (usually liquid) and light solid particles. The bed is fluidized by a downflow of the continuous fluid. It is

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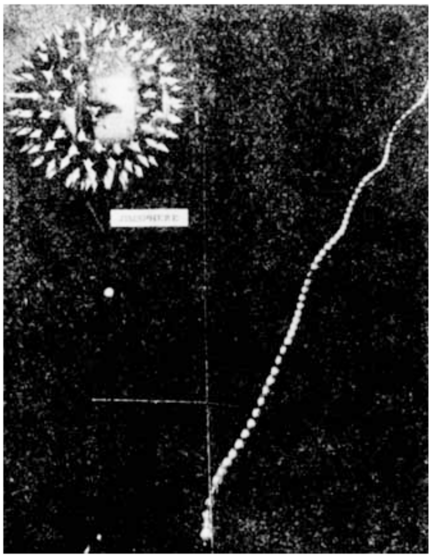

Figure 1. Trajectory of a rising meteorological balloon.
Courtesy of the American Geophysical Union (*J. of Geophys. Res.*, **69**, 591, 1964).

used mainly in biotechnology as the core of a new class of bioreactors (Nikolov and Karamanev, 1987). These studies showed unusual behavior of the inverse fluidized bed when compared to that of the classic fluidized bed where the particle density is greater than that of the fluid. The most important result of the hydrodynamic experiments was that the terminal velocity of light particles, determined indirectly from the bed expansion data, differs from that calculated from Eq. 1 and the standard drag curve (Eq. 2) by up to 50% when Re_t was greater than approximately 130. Similar results were reported by Garnier et al. (1990). However, the terminal velocity obtained for the light particles with $Re_t < 130$ was in good agreement with the theoretical predictions.

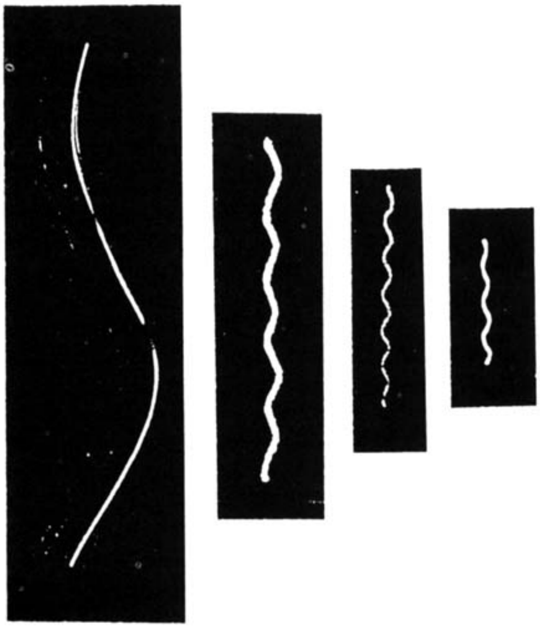

Figure 2. Typical trajectories of free-rising solid spheres.

These results and the lack of data in the literature about hydrodynamics of freely rising light particles were the major reason for initiating the present work. The main aim of this note is to determine the velocity and the drag coefficient of freely-rising light spheres, and to compare these data with that predicted for the case of free settling.

Materials and Methods

The experiments for determination of the free-rising velocity of light particles were conducted in a plexiglass column (diameter of 0.25 m and a height of 1.9 m) filled with tap water (temperature 23°C). The column diameter was large enough to prevent wall effects on the particle movement (Filderis and Whitmore, 1961). The particles were delivered to the bottom of the column by means of a special clamp and after a 5-min period (necessary for liquid to become quiescent), they were released. The steady-state particle rising velocity was measured using a photographic Nikon FG camera with an exposure time of 1, 1/2 and 1/4 s. The experiments were performed in a dark room. The particles were illuminated by a 100-W reflection lamp. The particle trajectory during the exposure time was thus captured on the negative film. The terminal velocity was calculated by dividing the length of particle trajectory in the vertical direction by the exposure time. The shape of the trajectory was determined directly from the photographic picture. The error of the velocity measurements did not exceed 2.5%.

The solid particles were made of expanded polystyrene (styrofoam), hollow plexiglass, hollow and solid polyethylene, paraffin, and cork. They all had a spherical shape with diameters varying from 1.5 to 80 mm. The densities were between 30 and 990 kg/m^3, and the terminal Reynolds numbers were in the range 2.5×10^1–9×10^4.

The particle diameters were measured by a micrometer. Their density was determined in three different ways:

1) The density of large spheres ($d > 1$ cm) was measured by dividing their mass (measured by a Sartorius Excellence balance with a precision of 0.1 mg) by their volume calculated from the diameter.

2) The density of smaller spheres was measured by a pycnometer filled with distilled water at a temperature between 30.0 and 30.1°C. The mass of these particles was measured by a Sartorius Supermicro S4 balance with a precision of 0.001 mg.

3) When the particle density was between 850 and 990 kg/m^3, the exact density was measured by preparing a water-ethyl alcohol solution with a concentration such that the particle remained neutral in this solution (neither rising nor settling) for longer than 15 s. In addition, the density of the solution was measured by a pycnometer. The particle density was considered to be equal to that of the solution.

Results

All the particles with $Re_t > 130$ and $\rho_p < 300$ kg/m^3 rose by spiral trajectory (Figure 2). Almost every spiral had a near constant diameter and wavelength. The ratio between the spiral wavelength and its diameter is given in Figure 3. This ratio remained constant in the range $500 < Re_t < 9 \times 10^4$, and there-

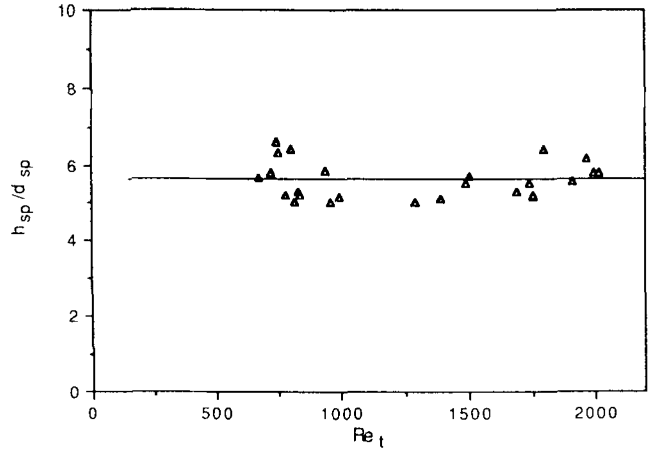

Figure 3. Dependence between h_{sp}/d_{sp} and Re_t.

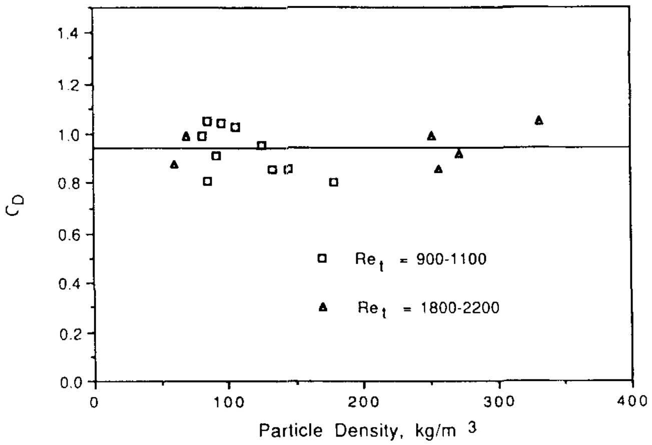

Figure 5. Effect of particle density on the drag coefficient.

fore all the spirals were geometrically identical. Hence, the angle between the particle velocity vector and the horizontal plane was constant and equal to 61°.

The drag coefficient was determined from Eq. 1 rewritten in the following form:

$$C_D=\frac{4gd(\rho_l-\rho_p)}{3\rho_l U_t^2} \tag{3}$$

The experimental relationship between C_D and Re_t for the case of freely-rising particle is shown in Figure 4. It can be seen that when Re_t was less than 130, the drag coefficient of a light particle was the same as that of a heavy particle, and therefore the movement of the light particle can be described by the standard drag curve. However, with $Re_t>130$ and $\rho_p<300$ kg/m^3, a significant difference between the drag coefficients of the light and heavy particles was observed. In this region, C_D was almost constant and equal to $0.95\pm11\%$ which is more than twice that predicted by the Newton's law. The drag coefficients of light particles ($\rho_p>900$ kg/m^3) were also studied. The data obtained agreed well with the standard drag curve in the range $25<Re_t<3{,}000$ (Figure 4).

Therefore, the drag curve obtained for the free-rising light particle can be described by the following system of equations:

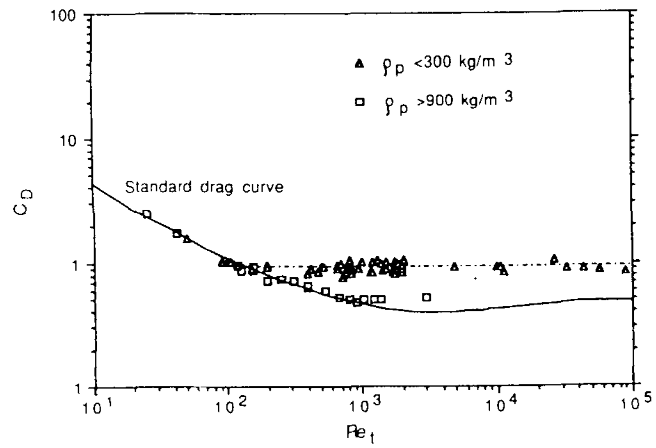

Figure 4. Dependence between C_D and Re_t.

$$C_D=\frac{24(1+0.173Re_t^{0.657})}{Re_t}+\frac{0.413}{1+16{,}300Re_t^{-1.09}} \tag{4a}$$

for $Re_t<130$ and/or $\rho_p>900$ kg/m^3;

$$C_D=0.95 \tag{4b}$$

for $130<Re_t<9\times10^4$ and $\rho_p<300$ kg/m^3.

The effect of particle density on C_D at constant Reynolds numbers is shown in Figure 5. It can be seen that when $\rho_p<300$ kg/m^3, the average drag coefficient is equal to 0.95 and is independent of the particle density.

Discussion

The specific behavior of free-rising particles can be explained by the effect of turbulence on the particle movement (Hetsroni, 1989). The fluid flow around a moving spherical particle has been studied (Fan and Tsutchiya, 1990; Clift et al., 1978). This flow is viscous when $Re<1$. At Reynolds numbers between 1 and 130, a stable wake formation behind the sphere is observed. Both these regions can be characterized by the axial symmetry of the flow streamlines. Therefore, all nonvertical forces applied to the particle are balanced, and the free-falling and -rising spheres have linear vertical trajectories and equal C_D. As Re increases above 130, a wake shedding begins. Periodic pulsations of the fluid around the sphere are observed (Fan and Tsutchiya, 1990; Batchelor, 1967); hence, the flow streamlines are not axisymmetric in this region. This causes an imbalance of the forces applied to the sphere in nonvertical direction. The main force (along with that of fluid viscosity) opposing these nonvertical forces is that of mechanical inertia of the spherical particle. The main difference among a light and a heavy particle with the same driving forces, but with opposite directions, equal absolute values of $(\rho_p-\rho_l)$, and equal diameters is that the light particle is less inert because of its smaller density and therefore smaller mass. When $|\rho_p-\rho_l|$ is small, both the heavy and light particles have similar behavior, since their mechanical inertia is similar. It can be shown that the light particle density and therefore its inertia are smaller than those of heavy particle, by less than 20%, when $|\rho_p - \rho_l| < 100$ kg/m^3 (light particle density higher than 900 kg/m^3).

Conclusions

The main conclusions which can be drawn from these results are:

1) The drag coefficient of free-rising light particles is a constant, equal to 0.95 when $\rho_p < 300$ kg/m^3 and $Re_t > 130$.

2) C_D of free-rising light particle can be described by the laws of free settling, only when $Re_t > 130$ and/or ρ_p is greater than approximately 900 kg/m^3.

3) C_D of free-rising light particle depends on both Re_t and ρ_p when $300 < \rho_p < 900$ kg/m^3 and $Re_t < 130$. Additional experiments are necessary for determination of this dependence.

Acknowledgment

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Notation

C_D = particle drag coefficient
d = particle diameter
g = acceleration due to gravity
Re_t = particle Reynolds number defined by $dU_t\rho_l/\mu$
U_t = terminal velocity of solid particle
μ = liquid viscosity
ρ_l = liquid density
ρ_p = particle density

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